# Method of Slices(分割法)

LA, SLM can be applied only to limited conditions, e.g., uniform or simple ground formation and simple ground water conditions, but they can be hardly applied for multiple layered soils and complicated ground water conditions.

Method of slices (one of LEMs) is one of the most commonly used methods for evaluating the stability of geotechnical structures with complicated conditions, especially for slope stability, from which factor of safety ($F_s$) is estimated.

<u>Procedure of method of slices</u>

1. Assuming trial slip surface
2. Dividing slipping block into vertical slices
3. Force and moment equilibrium for each slice and overall moment and force equilibrium
4. Factor of safety on shear strength of the trial slip surface
5. Finding minimum factor of safety



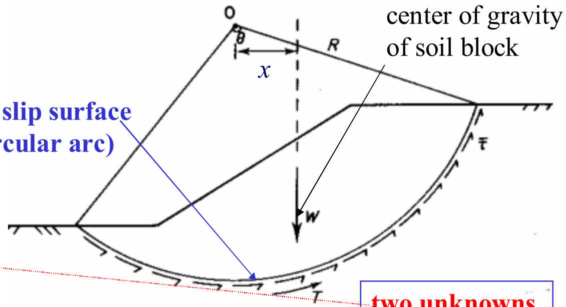
Various methods have been proposed in this type of stability analysis.

- *Swedish (Fellenius') method*
- *Bishop's method*
- *Jambu's method*
- *Spencer's method*
- *Mogenstern and Price's method*

Difference of these methods:
•shape of slip surface (circular or non-circular)
•<u>assumption on interslice force</u>（スライス間力の仮定）

Why this assumption is needed? ⇨ to determine the normal force on slip surface of each slice.

## $\phi_u$=0 method




Soil properties: $c_u$,$\gamma$
Length of arc: $L=R\theta$
$\bar{\tau}$: average shear stress along $L$
$T=\bar{\tau}L$ (1)
$W$: weight of soil block

**two unknowns**
**two equations**

***statically determinate***

Moment about O: overturning moment $M_d=Wx$ (2)
resisting moment $M_r=TR$ (3)
Failure criteria: $\tau_f=c_u$ **(4)**
Mobilized shear strength $\tau=\tau_f/F_s$ ($F_s$: Factor of safety) (5)
In equilibrium $Wx=TR$ **(6)**

from eqs. (1) ,(4) ,(5) and (6)

$$F_s=\frac{c_u LR}{Wx} \qquad (7)$$

This method can be adopted to the conditions:
varying shear strength,
presence of surcharge and water at the toe.



## Ordinary method of slices: Swedish or Fellenius' method

**In $\phi_u$=0 analysis, undrained shear strength on slip surfaces can be assumed to be independent of the stress level. In effective stress analysis (or $\phi$>0 material), the shear strength on the slip surface is function of the (effective) normal stress (by Mohr-Coulomb failure criteria) and thus the normal stress along the failure surface must be determined or taken into account in the analysis*). This may be achieved by dividing the failure mass into a number of slices.**

**The Swedish (Fellenius') method is the simplest method of slices. In this method the normal force on the base of each slice is determined by considering the equilibrium of forces normal to the base. To make the problem determinate, the assumption is made that the resultant to the interslice forces acting on any slice is parallel to its base.**

***) Beside method of slices, friction circle method can be applied to uniform soil conditions (c,$\phi$,$\gamma$:const) but not to the soil with varying properties.**

# Swedish (Fellenius') method

Soil properties: $c', \phi', \gamma$

•For a slice at its base with
normal stress $\sigma$,
shear stress $\tau$ and
pore pressure $u$

-Failure criteria: $\tau_f = c' + (\sigma - u)\tan\phi'$ **(1)**

-Mobilized shear strength $\tau = \tau_f / F_s$

since $P = \sigma l$, $T = \tau l$,

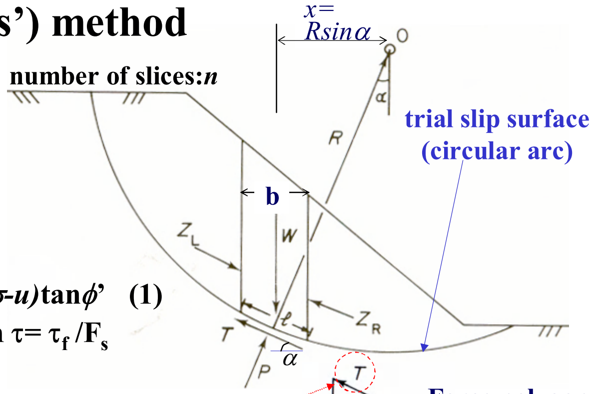
$$T = \frac{1}{F_s}(c'l + (P - ul)\tan\phi') \quad \textbf{(2)}$$

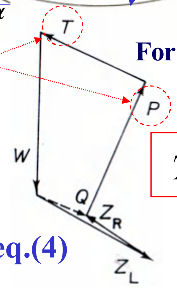
**$2n$ unknowns +1($F_s$)**

-Assuming $\vec{Q}(=\vec{Z}_L - \vec{Z}_R)$ is parallel to the base of slice and solving equilibrium normal to the base,

***P=Wcosα* (3)**

**$2n$ equations +1 (eq.(4))**




$\vec{T} // \vec{Q}$



**•Overall Moment equilibrium:**

Note: interslice forces are internal and their net moment is zero.

$$\sum W_i R \sin\alpha_i = \sum T_i R \quad (4)$$

from eq.(2) $$\sum W_i \sin\alpha_i = \sum \frac{1}{F_s}(c'l_i + (P_i - u_i l_i)\tan\phi') \quad (5)$$

hence $$F_{sm} = \frac{\sum(c'l_i + (P_i - u_i l_i)\tan\phi')}{\sum W_i \sin\alpha_i} \quad (6)$$

**factor of safety satisfying overall moment equilibrium**

Neglecting interslice force
+
***l=b/cosα***

using eq.(3) $$F_{sm} = \frac{\sum(c'l_i + (W_i\cos\alpha_i - ul_i)\tan\phi')}{\sum W_i \sin\alpha_i} \quad (7)$$

***liner equation about $F_S$ :easy to be solved***

**overestimating the effect of pore pressure, underestimating effective stress.**

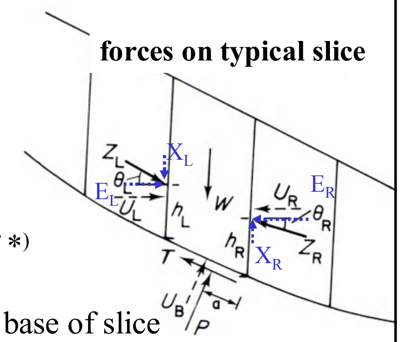
***large error***

This assumption implicitly makes the problem *statically determinate*.

$W_i' \cos\alpha_i$

$$F_{sm} = \frac{\sum(c'l_i + W'_i\cos\alpha_i \tan\phi')}{\sum W_i \sin\alpha_i} \quad (8)$$ ← ***Modified Fellenius' Method***

## Equations and Unknowns in MS




**For sliding mass divided into *n* slices**

•Equations available: total $3n$ (V, H, M equilibrium)

•Unknowns:
- $1$ : $F_s$ relating shear forces $T$ *) to normal forces $P$
- $n$ : Normal total forces $P$ on base of slice (pore water forces $U_B$ are known)
- $n$ : Positions $a$ of forces $P$
- $n$-1 : Interslice total forces $Z$ (pore water forces $U_L$, $U_R$ are known)
- $n$-1 : Inclinations $\theta$ of interslice forces
- $n$-1 : Height $h$ of inter slice forces

total $5n$-2 **$2n$-2**: statically indeterminate

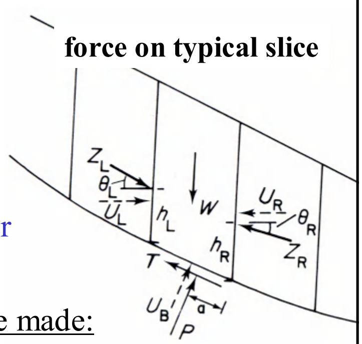
Z and θ can be replaced by horizontal and vertical components: $X$ and $E$.

*)failure criteria and T can be considered as given equation and unknowns



## General Formulation on Limit Equilibrium on MS

### - Equations and Unknowns in MS -




Indeterminate problem is better to be solved using compatibility. But due to the assumption of rigid body, it is difficult to introduce the compatibility conditions.

Hence **$2n$-2** assumptions must be made for the problem to be statically determinate.

Several classes of assumption which may be made:

1. Assumptions about the distribution of normal stress along the slip surface.
2. Assumptions about the position of the line of thrust of the interslice forces.
3. Assumptions about the inclination of the interslice forces.

difference of various methods of slices ⇔ difference of the assumption

## Assumptions in method of slices

In most methods, $P$ is assumed to act at the center of the base of each slice (Class 1.). This assumption is reasonable providing the slices are thin, and reduces the number of required assumption to ***n*-2**.

In many methods, an assumption is made about the inclinations of the interslice forces (Class 3.). But this gives another ***n*-1** assumptions making the problem ***over-specified***. This analysis may then be carried out either satisfying **overall moment equilibrium** or **horizontal force equilibrium**, yielding two factors of safety, $F_{sm}$ and $F_{sf}$, which are generally different with this condition.

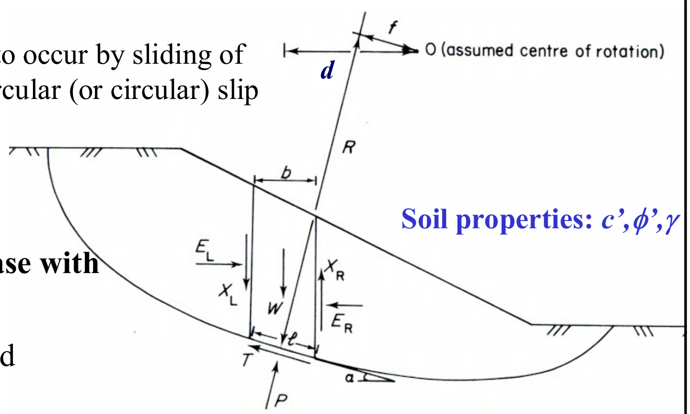
Fredlund and Krahn (1977) have shown the general equations of equilibrium. The formulation is the same for circular and no-circular slip surface, although for the latter a frictional center of rotation is adopted.

***difference of the methods***



## General Formulation on Limit Equilibrium on MS

Failure is assumed to occur by sliding of a block on a non-circular (or circular) slip surface.




**Soil properties: $c', \phi', \gamma$**

- **For a slice at its base with**
  - normal stress $\sigma$,
  - shear stress $\tau$ and
  - pore pressure $u$

-**Failure criteria**: $\tau_f = c' + (\sigma - u)\tan\phi'$ **(1)**

-**Mobilized shear strength** $\tau = \tau_f / F_s$

since $P = \sigma l$, $T = \tau l$, $T = \frac{1}{F_s}(c'l + (P - ul)\tan\phi')$ **(2)**

**-Vertical equilibrium condition**:

$$P\cos\alpha + T\sin\alpha = W - (X_R - X_L) \quad (3)$$

By substituting eqs.(2) into eq. (3) and rearranging,

$$P = \left[ W - (X_R - X_L) - \frac{1}{F_s}(c'l\sin\alpha - ul\tan\phi'\sin\alpha) \right] / m_\alpha \quad (4)$$

where $m_\alpha = \cos\alpha\left(1 + \tan\alpha\frac{\tan\phi'}{F_s}\right)$

**-Horizontal equilibrium condition**:

$$T\cos\alpha - P\sin\alpha + E_R - E_L = 0 \quad (5)$$

By substituting eqs.(2) into eq. (5),

$$E_R - E_L = P\sin\alpha - \frac{1}{F_s}\left[(c'l + (P - ul)\tan\phi'\right]\cos\alpha \quad (6)$$



**•Overall Moment equilibrium (about O):(土塊全体のモーメントの釣合)**

$$\sum W_i d_i = \sum T_i R_i + \sum P_i f_i \quad (7)$$

By substituting eqs.(2) and (4) into eq.(7) and rearranging,

$$F_{sm} = \frac{\sum\left[c'l_i + (P_i - u_i l_i)\tan\phi'\right]R_i}{\sum(W_i d_i - P_i f_i)} \quad (8)$$

**factor of safety satisfying overall moment equilibrium**

For circular slip surfaces $f$=0, $d$=$R$sinα and $R$=cosnt, so

$$F_{sm} = \frac{\sum\left[c'l_i + (P_i - u_i l_i)\tan\phi'\right]}{\sum W\sin\alpha_i} \quad (9)$$

Eqs. (8) and (9) are ***nonliner equations*** about $F_{sm}$, because $P$ includes $F_s$. ***See eq.(4).***
Their solutions necessitate an iterative procedure.

•**Overall Force equilibrium: (土塊全体の力の釣合)**

In the absence of surface loading,

$$\sum(E_{Ri}-E_{Li})=0$$
$$\sum(X_{Ri}-X_{Li})=0 \qquad (10)$$

from eq.(6)

$$\sum E_{Ri}-E_{Li}=\sum P_i\sin\alpha_i-\sum\frac{1}{F_{sf}}[(c'l_i+(P_i-u_il_i)\tan\phi']\cos\alpha_i=0 \qquad (11)$$

$$F_{sf}=\frac{\sum[(c'l_i+(P_i-u_il_i)\tan\phi']\cos\alpha_i}{\sum P_i\sin\alpha_i} \qquad (12)$$

**factor of safety satisfying overall horizontal force equilibrium**

In order to solve for $\boldsymbol{F_{sm}}$ and $F_{sf}$, $P$ must be evaluated, which requires evaluation of $\boldsymbol{X_R}$, $\boldsymbol{X_L}$, *the interslice shear forces,* included in eq.(4).



## Common assumptions on interslice shear forces

$X_R-X_L=0$ :Bishop (1955) +neglecting $F_{sf}$

Jambu's simplified(1956) +
neglecting $F_{sm}$ with non-circular slip surface

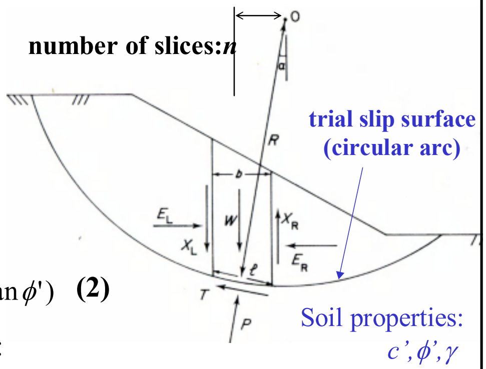
$\frac{X}{E}=\tan\theta=const.$ :Spencer (1967) + θ satisfying $F_{sm}$=$F_{sf}$

$\frac{X}{E}=\lambda f(x)$ :Mogenstern and Price (1965) +
for given *f(x)* the scaling factor λ is found satisfying $F_{sm}$=$F_{sf}$.

## Bishop simplified method of slices

$x = R\sin\alpha$

number of slices: $n$




trial slip surface (circular arc)

Soil properties: $c', \phi', \gamma$

- **For a slice at its base with $\sigma$, $\tau$ and $u$**
  - **-Failure criteria:**

    $\tau_f = c' + (\sigma - u)\tan\phi'$ **(1)**
  - **-Mobilized shear strength**

    $\tau = \tau_f / F_s$ since $P = \sigma l$, $T = \tau l$,

    $$T = \frac{1}{F_s}(c'l + (P - ul)\tan\phi') \quad \textbf{(2)}$$
  - **-Vertical equilibrium condition:**

    $$P\cos\alpha + T\sin\alpha = W - (X_R - X_L) \quad (3)$$

    Assuming $X_R - X_L = 0$ (i.e., interslice forces act only horizontally)

$$P = \left[W - \frac{1}{F_s}(c'l\sin\alpha - ul\tan\phi'\sin\alpha)\right] / m_\alpha, \quad m_\alpha = \cos\alpha\left(1 + \tan\alpha\frac{\tan\phi'}{F_s}\right) \quad (4)$$



*neglecting overall force equilibrium*

- **Overall Moment equilibrium (about O):**

$$\sum W_i R\sin\alpha_i = \sum T_i R \quad (5)$$

from eq.(2) $$\sum W_i\sin\alpha_i = \sum\frac{1}{F_s}(c'l_i + (P_i - u_i l_i)\tan\phi') \quad (5)$$

hence $$F_{sm} = \frac{\sum(c'l_i + (P_i - u_i l_i)\tan\phi')}{\sum W_i\sin\alpha_i} \quad (6)$$

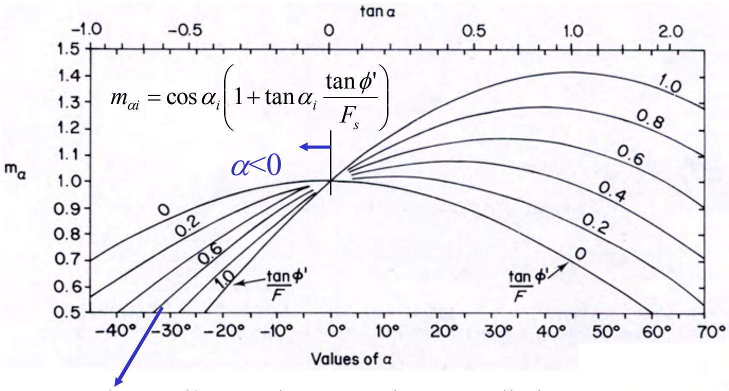
substituting eq.(4) into eq.(6) and using $b = l_i\cos\alpha_i$

$$F_{sm} = \frac{\sum\left[(W_i - ub_i)\tan\phi') + cb_i\right] / m_{\alpha i}}{\sum W_i\sin\alpha_i}, \quad m_{\alpha i} = \cos\alpha_i\left(1 + \tan\alpha_i\frac{\tan\phi'}{F_s}\right) \quad (7)$$

In Bishop method, overall horizontal equilibrium is not satisfied. Bishop discussed that $F_s$ is not particularly sensitive to the interslice shear force providing overall equilibrium moment is satisfied. But it is not true for the case with large rotation of principal stress along the slip plane.

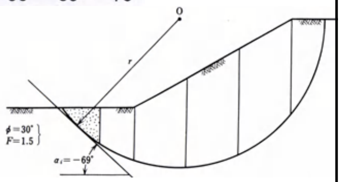
## Relationship between factor $m_\alpha$ and inclination of slice base

Anderson & Richards Slope Stability, Willy (1986)




Extremely small $m_\alpha$ value may give unrealistic $F_s$ in the case with negative base α angle. ( ex: tanϕ'=30°, Fs=1.5 and $\alpha$=-69°, $m_{\alpha i} \sim 0$)

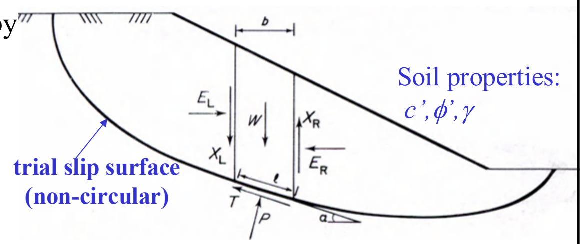
If there is a slice with small $m_\alpha$ (for example, less than 0.5), it better neglect the slice in the calculation.






## Jambu's simplified method

Failure is assumed to occur by sliding of a block on a ***non-circular*** slip surface.




- **For a slice at its base with σ, τ and *u***
  - **-Failure criteria:**

$$\tau_f = c' + (\sigma - u)\tan\phi' \quad (1)$$

  - **-Mobilized shear strength**

$\tau = \tau_f / F_s$ since $P=\sigma l$, $T=\tau l$, $T = \frac{1}{F_s}(c'l + (P-ul)\tan\phi')$ **(2)**

  - **-Vertical equilibrium condition**:

$$P\cos\alpha + T\sin\alpha = W - (X_R - X_L) \quad (3)$$

Assuming $X_R - X_L = 0$ (i.e., interslice forces act only horizontally)

$$P = \left[W - \frac{1}{F_s}(c'l\sin\alpha - ul\tan\phi'\sin\alpha)\right] / m_\alpha, \quad m_\alpha = \cos\alpha\left(1+\tan\alpha\frac{\tan\phi'}{F_s}\right) \quad (4)$$

-**Equilibrium condition parallel to base of slice**:

$$T+(E_R-E_L)\cos\alpha=(W-(X_R-X_L))\sin\alpha \quad (5)$$

From the assumption $X_R-X_L=0$ and eq.(1),

$$E_R-E_L=W\tan\alpha-\frac{1}{F_s}[(c'l+(P-ul)\tan\phi']\sec\alpha \quad (6)$$

•**Overall Force equilibrium:** *neglecting overall moment equilibrium*

In the absence of surface loading, $\sum(E_{Ri}-E_{Li})=0 \quad (7)$

Hence from eq.(6),

$$\sum E_{Ri}-E_{Li}=\sum W_i\tan\alpha_i-\sum\frac{1}{F_{s0}}[(c'l_i+(P_i-u_il_i)\tan\phi']\cos\alpha_i=0 \quad (8)$$

$$F_{s0}=\frac{\sum[(c'l_i+(P_i-u_il_i)\tan\phi']\sec\alpha_i}{\sum W_i\tan\alpha_i} \quad (9)$$

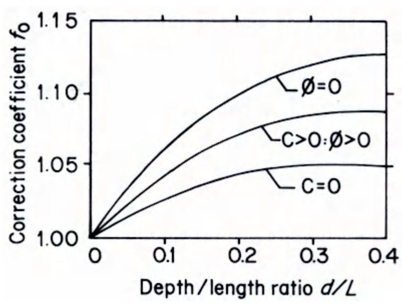
Eq.(9) is different from eq.(12) for general formulation, which comes from the neglect of interslice shear force.



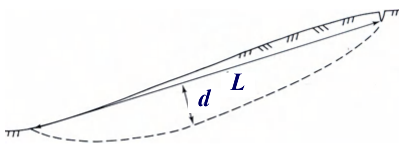
To take account of the interslice shear forces, Jambu et al. applied correction factor $f_0$ and gave the factor of safety $F_{sf}$ by the following equation.

$$F_{sf}=f_0F_{s0} \quad (10)$$

The correction factor $f_0$ was obtained by calibrating this analysis with Jambu's rigorous method*). The $f_0$ may be obtained from the figure below, depending on geometry of the problem as well as the soil conditions.



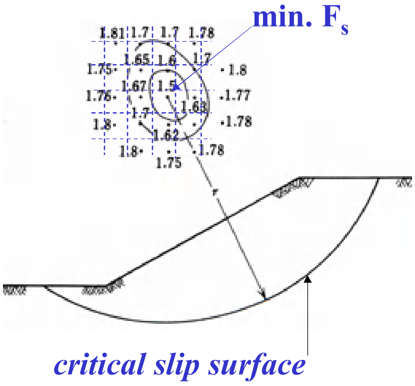

Anderson & Richards Slope Stability, Willy (1986)




*) In Jambu's rigorous method (1954), moment about the center of the base of each slice is taken into account, so that overall moment (implicitly) and force equilibrium are satisfied. For this, it is necessary to assume a position of the line of thrust of the interslice forces.

## Minimization of $F_s$

The equations on $F_s$ given above are for one arbitrary trial slip surface. As adopted in normal UBA and LEM, the slip surface which gives minimum $F_s$ is detected in a design. This is the factor of safety that should be used in the design with the given conditions.




For circular slip surfaces:

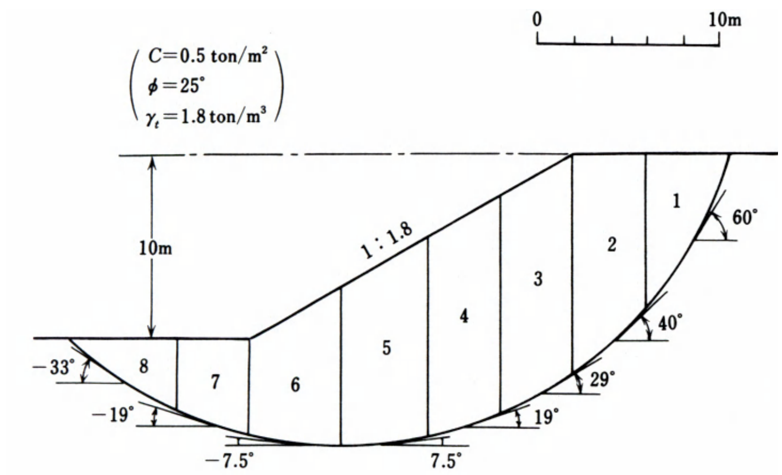
1. Find the circle giving min. $F_{si}$ about the node, $i$, of the mesh
2. Draw contours of min. $F_{si}$ .
3. Determine $F_s$ and critical slip surface

For non-circular slip surfaces:

Systematic procedure cannot be applied like circular surfaces, but try and error method should be used. For this reasons, no-circular slip line method is suitable for the condition where the possible slip plane has already been detected.



## Example of calculation of $F_s$



# Example of calculation of $F_s$ by Swedish method

| | b (m) | α (°) | l (m) | H (m) | cl (ton) | W* (ton) | W sin α (ton) | W cos α (ton) | W cos α ×tan φ (ton) | W cos α ×tan φ +cl |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4.5 | 60 | 9.0 | 4.8 | 4.5 | 38.88 | 33.67 | 19.44 | 9.06 | 13.57 |
| 2 | 4.0 | 40 | 5.22 | 10.2 | 2.61 | 73.44 | 47.20 | 56.26 | 26.23 | 28.84 |
| 3 | 4.0 | 29 | 4.57 | 12 | 2.29 | 86.4 | 41.89 | 75.57 | 35.24 | 37.53 |
| 4 | 4.0 | 19 | 4.23 | 11.6 | 2.12 | 83.52 | 27.19 | 78.97 | 36.82 | 38.94 |
| 5 | 5.0 | 7.5 | 5.04 | 10 | 2.52 | 90.0 | 11.75 | 89.23 | 41.61 | 44.13 |
| 6 | 5.0 | −7.5 | 5.04 | 7.4 | 2.52 | 66.6 | −8.70 | 66.03 | 30.79 | 33.31 |
| 7 | 4.0 | −19 | 4.23 | 5 | 2.12 | 36.0 | −11.72 | 34.04 | 15.87 | 17.99 |
| 8 | 6.0 | −33 | 7.15 | 2.5 | 3.58 | 27.0 | −14.71 | 22.64 | 10.56 | 14.14 |
| | | | | | | | ↓ Σ=126.57 | | | ↓ Σ=228.45 |

* $W=\gamma_t bH$

$\therefore F=\dfrac{228.45}{126.57}=1.8$



# Example of calculation of $F_s$ by Bishop's method

first approximation of $F_s$

| Slice No. | cb (ton) | W tan φ (ton) | Δ= W tan φ +cb (ton) | F=1.80 $m_\alpha$ | F=1.80 $\Delta/m_\alpha$ | F=2.15 $m_\alpha$ | F=2.15 $\Delta/m_\alpha$ | F=2.17 $m_\alpha$ | F=2.17 $\Delta/m_\alpha$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.25 | 18.13 | 20.38 | 0.724 | 28.14 | 0.688 | 29.63 | 0.686 | 29.70 |
| 2 | 2.0 | 34.25 | 36.25 | 0.933 | 38.85 | 0.905 | 40.03 | 0.904 | 40.09 |
| 3 | 2.0 | 40.29 | 42.29 | 1.00 | 42.29 | 0.980 | 43.16 | 0.979 | 43.21 |
| 4 | 2.0 | 38.95 | 40.95 | 1.030 | 39.76 | 1.016 | 40.30 | 1.015 | 40.33 |
| 5 | 2.5 | 41.97 | 44.47 | 1.025 | 43.39 | 1.020 | 43.61 | 1.020 | 43.62 |
| 6 | 2.5 | 31.06 | 33.56 | 0.958 | 35.03 | 0.963 | 34.84 | 0.963 | 34.84 |
| 7 | 2.0 | 16.79 | 18.79 | 0.861 | 21.82 | 0.875 | 21.44 | 0.876 | 21.46 |
| 8 | 3.0 | 12.59 | 15.59 | 0.698 | 22.34 | 0.721 | 21.64 | 0.722 | 21.59 |
| | | | | | ↓ Σ=271.62 | | ↓ Σ=274.65 | | ↓ Σ=274.84 |

1st trial : $F=\dfrac{271.62}{126.57}=2.15$

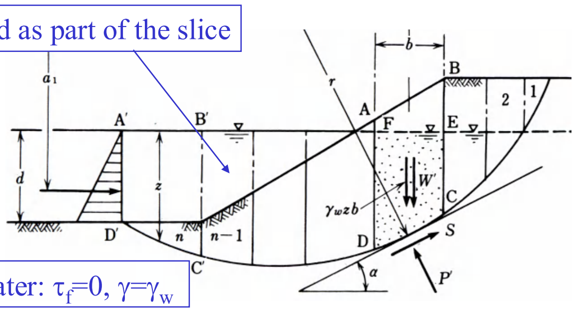
2nd trial : $F=\dfrac{274.65}{126.57}=2.17$

3rd trial : $F=\dfrac{274.84}{126.57}=2.17$

*convergence in iteration is quite good.*

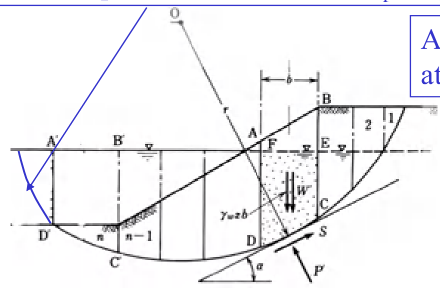
## Modeling of water in the analysis.

Water mass is considered as part of the slice

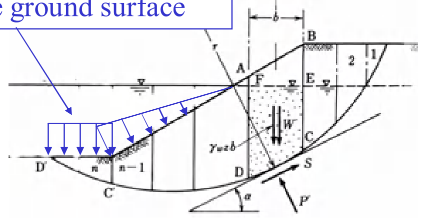



***Three methods give the same results.***

extend slip surface in the water: $\tau_f=0$, $\gamma=\gamma_w$



Appling water pressure at the ground surface





# Accuracy of LEM (MS)

**•Comparing the solution of LEM with the solution of other methods.**

**•Comparing the solutions of LEM (MS) each other.**

**Accuracy depends**

**-assumptions used in the analysis**

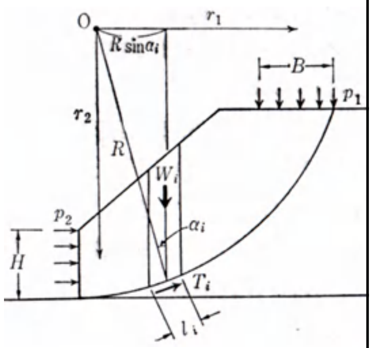
**-given conditions**

**(soil properties, geometry, boundary conditions)**

# Application of MS to stability problems

•**From overall Moment equilibrium (about O):**

$$p_1 B r_1 - p_2 H r_2 + R\sum W_i \sin\alpha_i = R\sum T_i$$
$$= \frac{R}{F_{sm}}\sum \tau_f l_i$$




Not only the Factor of safety in the ***slope stability*** problem:

•***Bearing capacity*** can be obtained by solving the above equation about $p_1$ with $F_{sm}$=1:

•Active ***earth pressure*** can be obtained by solving the equation about $p_2$ with $F_{sm}$=1.



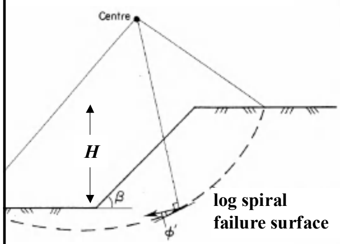



Table 2.4 Comparison of Stability Number $N=\gamma H/c$ by Methods of Limit Equilibrium and Limit Analysis (*Reproduced from Taylor, 1948; Chen, 1975, by permission of Elsevier Science Publishers*)

| | | Limit equilibrium | | | limit analysis |
|---|---|---|---|---|---|
| Slope angle β (°) | Friction angle φ (°) | slices | φ circle | logspiral | logspiral |
| 90 | 0 | 3.83 | 3.83 | 3.83 | 3.83 |
| | 5 | 4.19 | 4.19 | 4.19 | 4.19 |
| | 15 | 5.02 | 5.02 | | 5.02 |
| | 25 | 6.06 | 6.06 | 6.06 | 6.06 |
| 75 | 0 | 4.57 | 4.57 | 4.57 | 4.56 |
| | 5 | 5.13 | 5.13 | | 5.14 |
| | 15 | 6.49 | 6.52 | | 6.57 |
| | 25 | 8.48 | 8.54 | | 8.58 |
| 60 | 0 | 5.24 | 5.24 | 5.24 | 5.25 |
| | 5 | 6.06 | 6.18 | 6.18 | 6.16 |
| | 15 | 8.33 | 8.63 | 8.63 | 8.63 |
| | 25 | 12.20 | 12.65 | 12.82 | 12.74 |
| 45 | 0 | 5.88 | 5.88* | 5.88* | 5.53* |
| | 5 | 7.09 | 7.36 | | 7.35 |
| | 15 | 11.77 | 12.04 | | 12.05 |
| | 25 | 20.83 | 22.73 | | 22.90 |
| 30 | 0 | 6.41* | 6.41* | 6.41* | 5.53* |
| | 5 | 8.77* | 9.09* | | 9.13* |
| | 15 | 20.84 | 21.74 | | 21.69 |
| | 25 | 83.34 | 111.1 | 125.0 | 119.93 |
| 15 | 0 | 6.90* | 6.90* | 6.90* | 5.53* |
| | 5 | 13.89* | 14.71* | 14.71* | 14.38* |
| | 10 | | 43.62 | | 45.49 |

*Critical failure surface passes below toe.

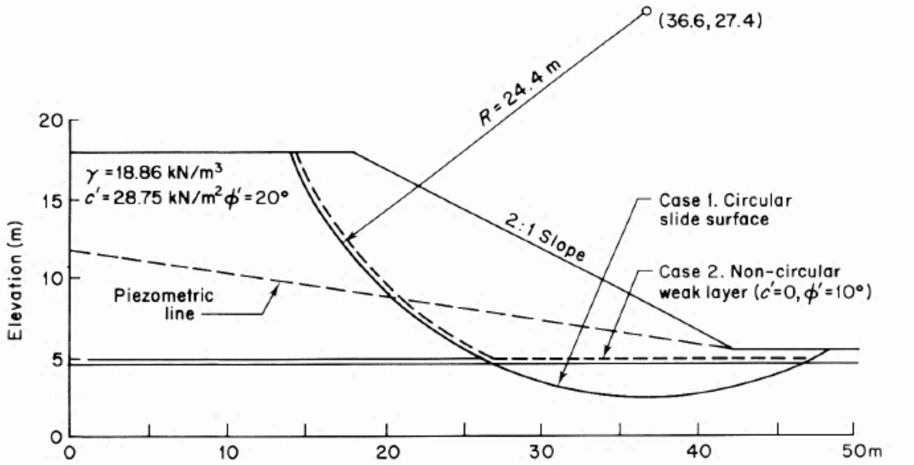



Figure 2.40 Example problem using circular and non-circular sliding surfaces. (*Adapted from Fredlund and Krahn, 1979, by permission of the National Research Council of Canada.*)



# Comparison of Fs for the example problem

Table 2.5 Comparison of Factors of Safety for Example Problem (*Reproduced from Fredlund and Krahn, 1977, by permission of the National Research Council of Canada.*)

| Case no. | Example problem* | Ordinary method | $F_{sm}$ Simplified Bishop method | Spencer's method F | Spencer's method θ | Spencer's method λ | $F_{sf}$ Janbu's simplified method | Janbu's rigorous method[†] | Morgenstern–Price method f(x) = constant F | Morgenstern–Price method f(x) = constant λ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Simple 2:1 slope, 12 m high, $\phi'=20°$, $c'=28.75$ kPa | 1.928 | 2.080 | 2.073 | 14.81 | 0.237 | 2.041 | 2.008 | 2.076 | 0.254 |
| 2 | Same as 1 with a thin, weak layer with $\phi'=10°$, $c'=0$ | 1.288 | 1.377 | 1.373 | 10.49 | 0.185 | 1.448 | 1.432 | 1.378 | 0.159 |
| 3 | Same as 1 except with $r_u=0.25$ | 1.607 | 1.766 | 1.761 | 14.33 | 0.255 | 1.735 | 1.708 | 1.765 | 0.244 |
| 4 | Same as 2 except with $r_u=0.25$ for both materials | 1.029 | 1.124 | 1.118 | 7.93 | 0.139 | 1.191 | 1.162 | 1.124 | 0.116 |
| 5 | Same as 1 except with a piezometric line | 1.693 | 1.834 | 1.830 | 13.87 | 0.247 | 1.827 | 1.776 | 1.833 | 0.234 |
| 6 | Same as 2 except with a piezometric line for both materials | 1.171 | 1.248 | 1.245 | 6.88 | 0.121 | 1.333 | 1.298 | 1.250 | 0.097 |

*Width of slice is 0.3 m and the tolerance on the non-linear solutions is 0.001

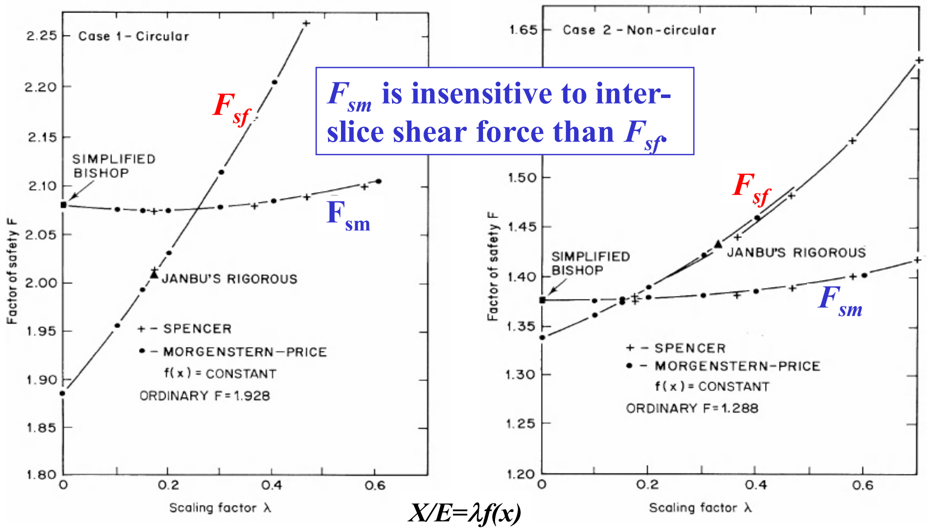
†The line of thrust is assumed at 0.333.

**Underestimate:** ***conservative***

***5 % dif.***

***Less than 0.4% difference***

## Influence of interslice forces on Factors of Safety



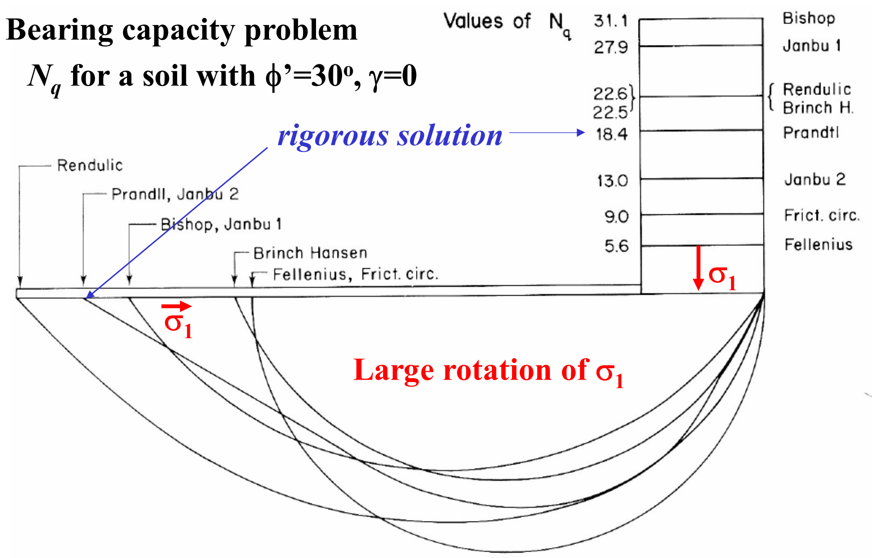

**Most commonly used *Simplified Bishop Method* can provide reliable answers similar to more rigorous ones.**



## Comparison of $N_q$ obtained from various methods

**Bearing capacity problem**

**$N_q$ for a soil with $\phi$'=30°, $\gamma$=0**



## Comparison of $N_q$ obtained from various methods

**Underestimate**

| Author | Method | Rupture Line | $N_q$ | % $N_{q_{Prandtl}}$ | $F = \frac{\tan 30°}{\tan\phi'_m}$ |
|---|---|---|---|---|---|
| Fellenius | Slices | Circle | 5.6 | 31 | 0.58 |
| Krey | Friction Circle | Circle | 9.0 | 49 | 0.75 |
| Janbu *et al.* | Slices | Prandtl | 13.0 | 71 | 0.88 × 1.05 = 0.92 |
| Prandtl | Plasticity | Prandtl | 18.4 | 100 | 1.0 |
| Brinch Hansen | Equilibrium | Circle | 22.5 | 113 | 1.07 |
| Rendulic | Extreme | Spiral | 22.6 | 114 | 1.07 |
| Janbu *et al.* | Slices | Circle | 27.9 | 151 | 1.15 × 1.05 = 1.21 |
| Bishop | Slices | Circle | 31.1 | 169 | 1.19 |

Figure 2.39 Comparison of bearing capacity factors determined by different methods. (*Adapted from Brinch Hansen, 1966, by permission of the Danish Geotechnical Institute.*)

**Overestimate**

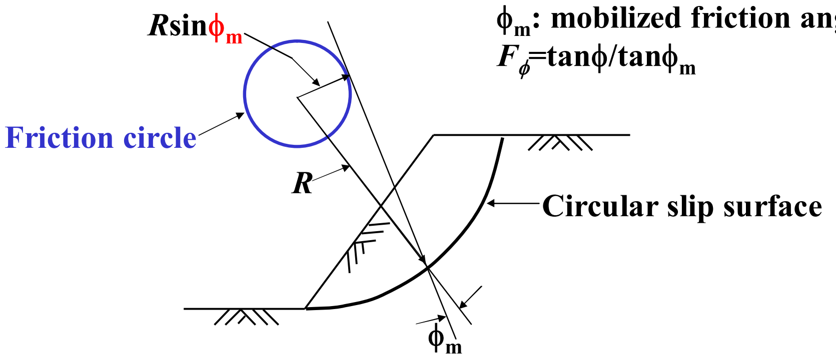
**If the slip surface is steeply inclined at the toe, a method should be chosen which gives a sensitive distribution of interslice forces.**



## Friction Circle Method

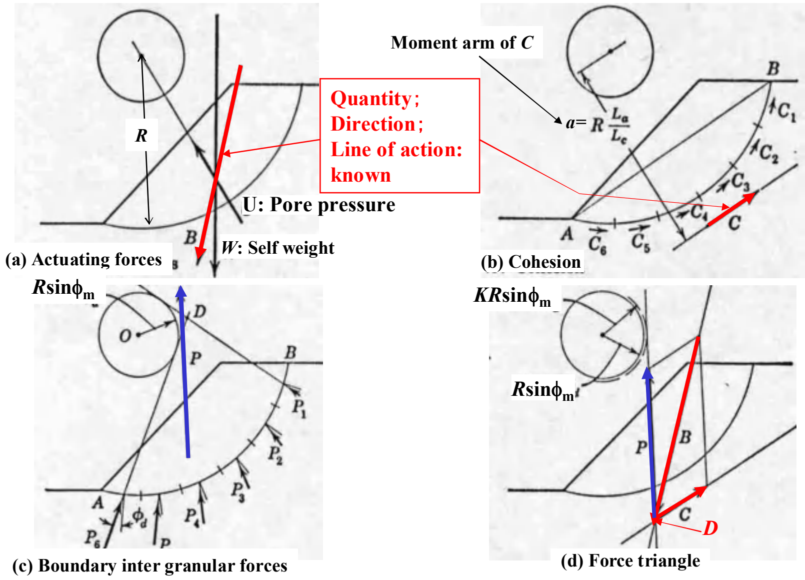
**Applicable to *uniform c-ϕ materials,***
***neither layered soil nor slope with seepage* .**
**Taylor's Stability Chart was made by FCM.**

**$\phi_m$: mobilized friction angle**
**$F_\phi = \tan\phi / \tan\phi_m$**



## Forces in stability analyses by Friction Circle Method

Taylor (1948)





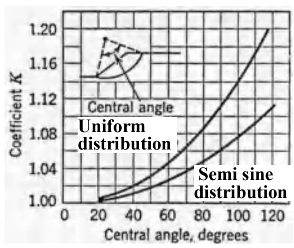

**$L_a$: length of arc AB, $L_c$:length of chord AB**

**Total force vector of cohesion:** $\vec{\mathbf{C}} \,//\, \overline{AB}, \left|\vec{\mathbf{C}}\right| = \dfrac{c}{F_c} L_c$

**From moment equilibrium:** $cL_a R = cL_c a \Rightarrow a = R\dfrac{L_a}{L_c}$

**Quantity; Direction; Line of action: known**

**Total force vector of friction forces *P* should be tangent to the circle with r=*KR*sin$\phi_m$ and pass the point D.**
**As K ~1 (right figure), *R*sin$\phi_m$ can be used in in frictional circle method.**




**In the use of Taylor's stability chart , the factor of safety of the slope is given by trail satisfying the following conditions.**

$$\tau = \frac{\tau_f}{F_s} = \frac{c + \sigma \tan\phi}{F_s} = \frac{c}{F_c} + \sigma\frac{\tan\phi}{F_\phi} \Rightarrow F_c = F_\phi = F_s$$